Problems to Week 6 Tutorial — MACM 101 (Spring 2025)

1. Let $A = \{1, \{1\}, \{2\}\}$. Which of the following statements are true?

a)	$1 \in A$	b)	$\{1\} \in A$
c)	$\{1\} \subseteq A$	d)	$\{\{1\}\} \subseteq A$
e)	$\{2\} \in A$	f)	$\{2\} \subseteq A$
g)	$\{\{2\}\} \subseteq A$		

2. For $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ let $A = \{1, 2, 3, 4, 5\}, B = \{1, 2, 4, 8\}, C = \{1, 2, 3, 5, 7\}, \text{ and } D = \{2, 4, 6, 8\}.$ Determine each of the following:

a)	$(A \cup B) \cap C$	b)	$A \cup (B \cap C)$
c)	$\overline{C}\cup\overline{D}$	d)	$\overline{C\cap D}$
e)	$(A \cup B) - C$	f)	$A \cup (B - C)$
g)	(B-C)-D	h)	B - (C - D)
i)	$(A \cup B) - (C \cap D)$		

Draw Venn diagrams for each of the expressions.

- 3. Demonstrate each of the following using Venn diagrams. (Assume a universe U.)
 - (a) If $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$.
 - (b) $A \subseteq B$ if and only if $A \cap \overline{B} = \emptyset$.
 - (c) $A \subseteq B$ if and only if $\overline{A} \cup B = U$.
- 4. Prove or disprove each of the following:
 - (a) For sets $A, B, C \subseteq U$, if $A \cup C = B \cup C$, then A = B.
 - (b) For sets $A, B, C \subseteq U$, if $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then A = B.
- 5. Prove that $A B = A \cap \overline{B}$.
- 6. Prove that $A\Delta B = (\overline{A} \cap B) \cup (A \cap \overline{B})$.
- 7. Investigate the truth or falsity of the following using 3 methods: Venn diagrams, laws of set theory, and proving that the left side is a subset of the right one and vice versa.

$$A - (B \cup C) = (A - B) \cap (A - C).$$