Problems to Week 7 Tutorial — MACM 101 (Spring 2025)

1. Let A, B, C, D be nonempty sets. Prove that $A \times B \subseteq C \times D$ if and only if $A \subseteq C$ and $B \subseteq D$.

2. Prove that $A \times (B - C) \subseteq (A \times B) - (A \times C)$. Does the equality holds?

3. Determine which of the following relations R on the set A are reflexive, symmetric, transitive, and anti-symmetric.

   (a) $A = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$. Draw the graph and the matrix of this relation.

   (b) A is the set of all students at SFU, and $(x, y) \in R$ means that the height of x differs from the hight of y by no more than one inch.

   (c) A is the set of ordered pairs of real numbers, that is, $A = \mathbb{R} \times \mathbb{R}$, and $((x_1, x_2), (y_1, y_2)) \in R$ if and only if $x_1 = y_1$ and $x_2 \leq y_2$.

   (d) Prove that the following relation is an order on the Cartesian product $\{1, 2, 3\} \times \{1, 2, 3\}$ and draw its diagram:

   $$((x_1, x_2), (y_1, y_2)) \in R \quad \text{if and only if} \quad x_1 < y_1, \text{ or } x_1 = y_1 \text{ and } x_2 \leq y_2.$$

   (Such an order is called the *lexicographic order*.)

   (e) Prove that the following relation on the set of all nonempty subsets of $\{a, b, c, d\}$ is an order, draw its diagram, find all the maximal, minimal, least and greatest elements:

   $$(x, y) \in R \quad \text{if and only if} \quad x \subseteq y.$$

4. Check that the following relation R on the set A are equivalence relations, find their equivalence classes, the number of equivalence classes, and determine which equivalence class the element z belongs to.

   (a) Let A be the set of all possible strings of 3 or 4 letters in alphabet $\{A, B, C, D\}$, let $z = BCAD$, and let $(x, y) \in R$ if and only if x and y have the same first letter and the same third letter.

   (b) Let A be the power set of $\{1, 2, 3, 4, 5\}$, let $z = \{1, 2, 3\}$, and let $(x, y) \in R$ if and only if $x \cap \{1, 3, 5\} = y \cap \{1, 3, 5\}$.