Problems to Week 8 Tutorial — MACM 101 (Spring 2025)

1. Determine whether or not the following relations are functions. If a relation is a function, find its range.

   (a) $\{(x, y) \mid x, y \in \mathbb{Z}, \ y = x^2 + 7\}$, a relation from $\mathbb{Z}$ to $\mathbb{Z}$;

   (b) $\{(x, y) \mid x, y \in \mathbb{R}, \ y^2 = x\}$, a relation from $\mathbb{R}$ to $\mathbb{R}$.

2. For each of the following functions, determine whether it is one-to-one and determine its range.

   (a) $f \colon \mathbb{Z} \to \mathbb{Z}, \ f(x) = 2x + 1$;

   (b) $f \colon \mathbb{Q} \to \mathbb{Q}, \ f(x) = 2x + 1$;

   (c) $f \colon \mathbb{Z} \to \mathbb{Z}, \ f(x) = x^3 - x$;

3. Let $f \colon A \to B$ where $A = X \cup Y$ with $X \cap Y = \emptyset$. If $f|_X$ and $f|_Y$ are one-to-one, does it follow that f is one-to-one?

4. For each of the following functions $f \colon \mathbb{Z} \to \mathbb{Z}$, determine whether the function is onto. If the function is not onto, determine the range of f.

   (a) $f(x) = 2x - 3$;

   (b) $f(x) = x^2 + x$.

5. Let $f, g \colon \mathbb{R} \to \mathbb{R}$, where $g(x) = 1 - x + x^2$ and $f(x) = ax + b$. If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a, b.

6. Let $g \colon \mathbb{N} \to \mathbb{N}$ be defined by $g(n) = 2n$. If $A = \{1, 2, 3, 4\}$ and $f \colon A \to \mathbb{N}$ is given by $f = \{(1, 2), (2, 3), (3, 5), (4, 7)\}$, find $g \circ f$.

7. If $f \circ g$ is onto, does it follow that g is onto?

8. For each of the following functions $f \colon \mathbb{R} \to \mathbb{R}$, determine whether f is invertible, and, if so, determine f^{-1}.

   (a) $f = \{(x, y) \mid 2x + 3y = 7\}$;

   (b) $f = \{(x, y) \mid y = x^4 + x\}$.